Assignment 2 *

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Model Setup

Economy with Divisible Labor

Consider a standard one-sector stochastic growth model. A representative household chooses consumption c_t, investment i_t, and labor supply $h_t \in [0,1]$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t + A \log(1-h_t)\right].$$

Output is produced using a Cobb–Douglas technology:

$$y_t = \lambda_t k_t^{\theta} h_t^{1-\theta},$$

where k_t is capital, h_t is labor input, and λ_t is a technology shock. The resource constraint and capital accumulation equation are

$$c_t + i_t \leq \lambda_t k_t^{\theta} h_t^{1-\theta},$$

$$k_{t+1} = (1-\delta)k_t + i_t.$$

Following Hansen's exposition, the technology shock evolves as a first-order Markov process. A convenient reduced-form representation is

$$\lambda_{t+1} = \gamma \lambda_t + \varepsilon_{t+1},$$

where ε_{t+1} is i.i.d. and parameterized so that the unconditional mean of λ_t is equal to 1. In this economy labor is continuously adjustable. As a result, fluctuations in aggregate hours operate through the **intensive margin**.

Economy with Indivisible Labor

Hansen's key modification is to impose **indivisible labor**. In each period an individual can either work a fixed number of hours h_0 or not work at all. Intermediate hours choices are not available.

*Replication handout based on Hansen, G. D. (1985), "Indivisible Labor and the Business Cycle.". Please submit the assignment by April 7 to chengao0716@gmail.com.

To make the environment tractable, Hansen follows Rogerson (1984) and introduces employment lotteries. Let $\alpha_t \in [0,1]$ denote the probability of being employed in period t. [1]Aggregate hours are then given by

$$h_t = \alpha_t h_0.$$

Under this lottery interpretation, the representative agent's expected one-period utility can be written as

$$U(c_t, \alpha_t) = \log c_t + A\alpha_t \log(1 - h_0).$$

The aggregate resource constraint remains

$$c_t + i_t \leq \lambda_t k_t^{\theta} h_t^{1-\theta},$$

and capital still evolves according to

$$k_{t+1} = (1-\delta)k_t + i_t.$$

The economic interpretation is different from the divisible-labor case: labor fluctuations now operate primarily through changes in employment, that is, through the **extensive margin**.

Parameterization

Use Hansen's baseline calibration as the starting point for your replication:

Parameter	Symbol	Value
Discount factor	β	0.99
Capital share	θ	0.36
Depreciation rate	δ	0.025
Leisure weight	A	2
Shock persistence	γ	0.95
Fixed hours in indivisible case	h_0	0.53
Shock standard deviation	σ_ε	0.00712

Replication Tasks

1. **State the two model environments.** Write down the representative-agent problem for each economy. Clearly identify the objective function, resource constraint, capital law of motion, state variables, control variables, and the technology shock process.

2. **Implement the baseline calibration.** Use the parameter values above and explain the economic role of each parameter. *In particular, explain why the indivisible-labor model requires the additional parameter h_0.*

[1]Please do go back to the paper to see the detailed explanation of the employment lotteries.

3. **Solve the two economies.** You may follow Hansen's approximation strategy or use another numerical method, such as linearization, discretization, or a projection-based approach, *provided that the two economies are solved in a comparable way.* If your method differs from Hansen's, explain the difference.

4. **Simulate artificial time series.** For each model, generate artificial time series and compute the same class of statistics studied in the paper. A recommended benchmark is:

   - simulation length of 2000 periods,
   - 100 repeated simulations,
   - log transformation and HP filtering with smoothing parameter $\lambda = 1600$,
   - sample standard deviations and correlations with output.

5. **Replicate the main moments in Table 1.** Report the volatility and correlation with output for

$$y_t, \quad c_t, \quad i_t, \quad k_t, \quad h_t, \quad \text{productivity} = \frac{y_t}{h_t}.$$

   At a minimum, report the results for the divisible-labor and indivisible-labor economies and compare them with the target values in the paper.

6. **Explain the mechanism.** Use your simulated results to explain why the indivisible-labor model generates larger labor fluctuations and why the ratio of hours volatility to productivity volatility is much higher than in the divisible-labor economy.

Replication Targets

Your replication does not need to match the published numbers exactly digit by digit, but it should recover the same qualitative conclusions and similar magnitudes. The core target statistics from Hansen's Table 1 are:

Series	Divisible labor		Indivisible labor	
	Std. dev.	Corr. with y	Std. dev.	Corr. with y
Output	1.35	1.00	1.76	1.00
Consumption	0.42	0.89	0.51	0.87
Investment	4.24	0.99	5.71	0.99
Capital stock	0.36	0.06	0.47	0.05
Hours	0.70	0.98	1.35	0.98
Productivity	0.68	0.98	0.50	0.87

In your discussion, make sure to answer at least the following two questions:

- Why are hours substantially more volatile in the indivisible-labor economy?
- Why does the indivisible-labor model more easily generate hours fluctuations that are larger than productivity fluctuations?

Submission Instructions

- Submit a **Jupyter notebook** (`.ipynb`) that runs from top to bottom and reproduces your main results, tables, and figures.
- Submit a short **PDF or Markdown report** that explains the model setup, parameterization, solution method, and interpretation of the results.
- The report does not need to be long, but it should clearly explain how you moved from Hansen (1985) to your final replication output.

Reference

Hansen, G. D. (1985). Indivisible labor and the business cycle. *Journal of Monetary Economics*, 16(3), 309–327.

Rogerson, R. (1984). Indivisible labour, lotteries and equilibrium. Unpublished manuscript, University of Rochester.

Kydland, F. E. and Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica*, 50(6), 1345–1370.