Monetary Policy according to HANK

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1 Introduction

1.1 HANK: Heterogeneous Agent New Keynesian Models

- This paper: Kaplan, Moll, and Violante (2018)
- **Framework for quantitative analysis of the transmission mechanism of monetary policy**
- **Three Building Blocks:**
  1. Uninsurable idiosyncratic income risk
  2. Nominal price rigidities
  3. Assets with different degrees of liquidity

1.2 How Monetary Policy Works in RANK

- **Total consumption response to a drop in real rates:**

$$C \text{ response} = \underbrace{\text{direct response to } r}_{>95\%} + \underbrace{\text{indirect effects due to } Y}_{<5\%}$$

- **Direct response is everything**: pure intertemporal substitution

However, data suggest:

1. Low sensitivity of C to r
2. Sizable sensitivity of C to Y
3. Micro sensitivity is vastly **heterogeneous** and depends crucially on **household balance sheets**

1.3 How Monetary Policy Works in HANK

- **Once matched to micro data, HANK delivers realistic:**
  - Wealth distribution: small direct effect
  - MPC distribution: large indirect effect (depending on ΔY)

$$C \text{ response} = \underbrace{\text{direct response to } r}_{\text{RANK: }>95\%\text{ | HANK: }<1/3} + \underbrace{\text{indirect effects due to } Y}_{\text{RANK: }<5\%\text{ | HANK: }>2/3}$$

- **Overall effect depends crucially on fiscal response**, unlike in RANK where Ricardian equivalence holds.

2 Model Setup

2.1 Households

- A continuum of households is indexed by liquid assets b, illiquid assets a, and idiosyncratic productivity z
- Time is continuous and the aggregate state is the cross-sectional distribution $\mu_t(da, db, dz)$
- Productivity z follows an exogenous Markov process
- Households die at Poisson rate ζ; newborns enter with zero wealth and z drawn from the ergodic distribution

Households choose consumption c_t, labor supply ℓ_t, and deposits d_t into the illiquid account:

$$\max \; E_0 \int_0^\infty e^{-(\rho+\zeta)t} u(c_t, \ell_t)\, dt$$

where u is increasing and concave in consumption, and decreasing and convex in labor.

2.2 Household Balance Sheets

Liquid and illiquid assets evolve according to

$$\dot{b}_t = (1-\tau_t) w_t z_t \ell_t + r_t^b(b_t) b_t + T_t - d_t - \chi(d_t, a_t) - c_t$$

$$\dot{a}_t = r_t^a a_t + d_t$$

subject to

$$b_t \geq -\underline{b}, \qquad a_t \geq 0.$$

- Liquid borrowing pays the higher rate $r_t^{b-} = r_t^b + \kappa$
- Illiquid assets earn a higher return in equilibrium: $r_t^a > r_t^b$
- $d_t < 0$ denotes withdrawals from the illiquid account

2.3 Why Two Assets Matter

The transaction cost for moving resources across accounts is

$$\chi(d, a) = \chi_0 |d| + \chi_1 \left|\frac{d}{a}\right|^{\chi_2} a$$

- The linear term $\chi_0 |d|$ creates an inaction region
- The convex term ensures finite deposit and withdrawal rates
- This wedge generates households with very different liquid positions and MPCs
- Monetary policy therefore redistributes both through interest rates and through income changes

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2.4 Production Side

A representative final-good producer aggregates differentiated intermediate goods:

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

which implies demand for variety j:

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t, \qquad P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}.$$

Each intermediate producer has technology

$$y_{j,t} = k_{j,t}^{\alpha} n_{j,t}^{1-\alpha},$$

so real marginal cost is

$$m_t = \left(\frac{r_t^k}{\alpha} \right)^{\alpha} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha}.$$

2.5 Price Setting

Intermediate firms face Rotemberg price adjustment costs

$$\Theta_t(\pi_t) = \frac{\theta}{2} \pi_t^2 Y_t, \qquad \pi_t = \frac{\dot{P}_t}{P_t}.$$

This yields a continuous-time New Keynesian Phillips curve:

$$\left(r_t^a - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\varepsilon}{\theta} (m_t - m^*) + \dot{\pi}_t, \qquad m^* = \frac{\varepsilon - 1}{\varepsilon}.$$

- Inflation rises when markups are too low, i.e. when $m_t > m^*$
- Future profits are discounted at the illiquid return r_t^a

2.6 Illiquid Asset Composition

- The illiquid account contains physical capital and equity claims on intermediate-firm profits
- Let q_t be the share price and s_t the household's equity holdings, so

$$a_t = k_t + q_t s_t$$

- Since resources move freely within the illiquid account, no arbitrage implies

$$\frac{\Pi_t + \dot{q}_t}{q_t} = r_t^k - \delta =: \, r_t^a$$

- This lets the model collapse capital and equity into one composite illiquid asset with return r_t^a

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2.7 Monetary And Fiscal Policy

The central bank sets the nominal liquid return via a Taylor rule:

$$i_t = \bar{r}^b + \phi \pi_t + \epsilon_t, \qquad \phi > 1,$$

and the Fisher equation pins down the real liquid return:

$$r_t^b = i_t - \pi_t.$$

The government budget constraint is

$$\dot{B}_t^g + G_t + T_t = \tau_t \int w_t z \ell_t(a, b, z) \, d\mu_t + r_t^b B_t^g.$$

- Fiscal closure can come through taxes τ_t, transfers T_t, or spending G_t
- Because Ricardian equivalence fails, the fiscal response is quantitatively central in HANK

2.8 Equilibrium

An equilibrium consists of household choices, firm choices, prices, policy paths, and a distribution μ_t such that agents optimize and markets clear.

Key market-clearing conditions are:

$$B_t^h + B_t^g = 0, \qquad B_t^h = \int b\, d\mu_t$$

$$K_t + q_t = A_t, \qquad A_t = \int a\, d\mu_t$$

$$N_t = \int z\ell_t(a, b, z)\, d\mu_t$$

$$Y_t = C_t + I_t + G_t + \Theta_t + \chi_t + \kappa \int \max\{-b, 0\}\, d\mu_t$$

- The liquid-rate shock affects consumption directly through r_t^b
- In HANK, the larger amplification comes indirectly through wages, illiquid returns, and fiscal adjustment

3 Solution Method

3.1 Continuous-Time HA Models

- Following Achdou-Han-Lasry-Lions-Moll, solving a heterogeneous-agent model means solving a system of PDEs
- There are two core objects:
  1. A Hamilton-Jacobi-Bellman (HJB) equation for individual optimal choices
  2. A Kolmogorov Forward (KF) equation for the evolution of the cross-sectional distribution
- This apparatus is extremely general:
  1. Heterogeneous-household models: Bewley, Huggett, Aiyagari, HANK
  2. Heterogeneous-producer models: e.g. Hopenhayn-style environments
- A simple but powerful numerical method is the finite difference method

- See Benjamin Moll's continuous-time HA codes: http://www.princeton.edu/~moll/HACTproject_htm

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3.2 HJB: Individual Optimization

Let $V(a, b, z)$ denote the value of a household with illiquid assets a, liquid assets b, and productivity z.

In steady state, the HJB takes the form

$$(\rho+\zeta)V(a, b, z) = \max_{c,\ell,d} \Big\{u(c, \ell) + V_a(a, b, z)\big(r^a a + d\big) + V_b(a, b, z)\big((1-\tau)wz\ell + r^b(b)b + T - d - \chi(d, a) - c\big) + \mathscr{L}^z V + \zeta V_0(z$$

where $\mathscr{L}^z$ is the generator for idiosyncratic income risk.

- The HJB pins down policy rules for $c(a, b, z)$, $\ell(a, b, z)$, and $d(a, b, z)$
- Once we know these policies, we also know the drift of assets at each point in the state space

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3.3 FOCs And "Tomorrow Is Today"

Because time is continuous, the first-order conditions are local and look static.

- For liquid assets, the relevant shadow value is $V_b(a, b, z)$
- With separable CRRA utility, the consumption FOC is

$$u_c(c, \ell) = V_b(a, b, z)$$

and if $u(c, \ell)$ is CRRA in consumption, this becomes

$$c^{-\gamma} = V_b(a, b, z).$$

- This is what Moll means by "tomorrow is today"
- Instead of an expectation over a discrete next period, the HJB works with infinitesimal changes in the value function
- As a result, one can often compute the optimal controls directly from local derivatives of V

3.4 KF: Evolution Of The Distribution

Given optimal household policies, the cross-sectional distribution $g(a, b, z, t)$ evolves according to the Kolmogorov Forward equation

$$\partial_t g = -\partial_a\big(\dot{a}(a, b, z)g\big) - \partial_b\big(\dot{b}(a, b, z)g\big) + \mathscr{L}^{z*} g,$$

where $\mathscr{L}^{z*}$ is the adjoint of the income-shock generator.

- The KF equation is the law of motion for the distribution
- In steady state, it sets net probability flows equal to zero
- Solving the model therefore means finding:
  1. Policies that satisfy the HJB
  2. A distribution that is stationary under those policies

3.5 Finite Difference Method

- Put a grid on the state space for (a, b, z)
- Approximate derivatives like V_a and V_b with one-sided finite differences
- Use an upwind scheme so that the numerical derivative follows the direction of the endogenous drift
- This turns the continuous PDE problem into a sparse linear algebra problem

Schematic steady-state HJB system:

$$\rho v = u(v) + A(v)v$$

where $A(v)$ is the discretized generator implied by policies and drifts.

- Solving the Bellman equation becomes solving or iterating on a sparse matrix system
- After that, the distribution is recovered from the corresponding discretized KF equation

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3.6 Why Continuous Time Is Computationally Convenient

1. **Borrowing constraints are boundary conditions**

- In continuous time, inequality constraints show up at the boundary of the state space
- Interior FOCs still hold with equality
- This is cleaner than discrete time, where occasionally binding constraints directly distort Euler equations

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2. **Sparsity**

- Over an infinitesimal interval, a state can only move to nearby grid points
- In one dimension this often gives tridiagonal matrices
- In multiple dimensions the matrices are not literally tridiagonal, but they remain very sparse

3. **Tight link between economics and numerics**

- The drift of assets determines both the HJB discretization and the KF discretization
- The numerical objects have a direct economic interpretation as local probability flows

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3.7 Two Birds With One Stone

The HJB and KF problems are tightly linked.

- Discretize the HJB and obtain a generator matrix A
- Then the discretized KF equation uses the transpose of that same matrix:

$$\partial_t g = A^\top g$$

and in steady state

$$A^\top g = 0, \qquad \mathbf{1}^\top g = 1.$$

- Reason: the KF operator is the adjoint of the HJB operator
- So once the HJB is coded correctly, much of the distributional machinery comes "for free"
- This is one of the main computational insights in the Achdou et al. approach

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3.8 Aggregate Shocks

- Achdou et al. and the baseline HANK model here focus on idiosyncratic shocks only
- Aggregate shocks make the problem much harder because the entire distribution becomes a state variable
- A companion line of work develops efficient methods for this case:
  - "When Inequality Matters for Macro and Macro Matters for Inequality"
  - Open-source Matlab toolbox: https://github.com/gregkaplan/phact
- Conceptually, this extends linearization methods such as Campbell (1998) and Reiter (2009)
- Intuition: instead of a single global slope, the method tracks different local slopes at different points in the state space
- This is the bridge from continuous-time HA methods to Krusell-Smith-style environments with aggregate risk

References

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante. 2018. "Monetary Policy According to HANK." *American Economic Review* 108 (3): 697–743. https://doi.org/10.1257/aer.20160042.